

Hint for Question 3

Let $\epsilon > 0$ be arbitrary. since f_n is uniformly integrable, we can find a L such that $\sup_n \int f_n 1_{f_n > L} d\mu < \epsilon$. Write

$$\begin{aligned} \int f_n d\mu &= \int f_n 1_{f_n > L} d\mu + \int f_n 1_{f_n \leq L} d\mu \\ &\leq \epsilon + \int (f_n 1_{f_n \leq L} - L) d\mu + L\mu(\Omega). \end{aligned}$$

Now take limsup both sides and try to apply Fatou's lemma.