Homework 2.

Assigned Sept 15, 2005. Due Sept 27, 2005.

1. Let $\{B_t\}$ be a standard Brownian motion on some probability space. Let $\{t_n\}$ be a sequence of positive reals converging to 0 as $n \to \infty$. Recall that, we showed that $P(\limsup_{n\to\infty} \frac{B_{t_n}}{\sqrt{t_n}} = \infty)$ is either 0 or 1. Show that the probability is actually 1 by first arguing that the probability must be positive.

2. Let $\{B_t\}$ be a standard Brownian motion on some probability space. Let $\{\Pi_k\}$ be a sequence of partitions of $[0, T]$;

$$\Pi_k \doteq \{0 = t_1^k < t_2^k \cdots < t_{n_k}^k = T\}.$$

Suppose that $||\Pi_k|| \to 0$ as $k \to \infty$, where $||\Pi_k||$ denotes the mesh of the partition Π_k. Recall that we showed in class that

$$\sum_{i=1}^{n_k - 1} [W(t_{i+1}^k - W(t_i^k)]^2 \to T,$$

in L^2 as $k \to \infty$. Suppose now that $\sum_{k=1}^{\infty} ||\Pi_k|| < \infty$. Then show that the above convergence actually holds almost surely.
[**Hint: Use Borel Cantelli Lemma.**]