Homework 4.

Assigned Oct 6, 2005. Due Oct 20, 2005.

1. Let $M \in \mathcal{M}_2^c$ and $X \in \mathcal{L}_T^M$. Show that $\left\langle I^M(X), I^M(X)\right\rangle_t = \int_0^t X_u^2 d\left\langle M, M\right\rangle_u$ for all $t \in [0, T]$, a.s.

2. Let X, M be as in 1. Let $0 < a < b < T$. Show that $I_b^M(X) - I_a^M(X) = \int_0^T 1_{[a,b]}(t)X(t)dM(t)$.

3. Let $M, N \in \mathcal{M}_2^c$ and fix $X \in \mathcal{L}_T^M$ and $Y \in \mathcal{L}_T^N$. Show that

$$\left\langle I^M(X), I^N(Y)\right\rangle_t = \int_0^t X_u Y_u d\left\langle M, N\right\rangle_u,$$

for all $t \in [0, T]$, a.s.

4. Let $X, Y \in \mathcal{M}_{\text{loc}}^c$. Then there exists a unique adapted continuous process $\langle X, Y\rangle$ with bounded variation paths such that $XY - \langle X, Y\rangle$ is in $\mathcal{M}_{\text{loc}}^c$.

[*Wait until tuesday's class before attempting this problem.*]