

Practice Exam for Second Midterm Fall 2003

1. Let X and Y be independent random variables with means 5 and 6, respectively and variances 4 and 9, respectively. Find $E(4X - 6Y)^2$.
2. A professional basketball player has been a 70% free throw shooter. In the current season she made 300 of the 400 attempted free throws. How likely is this event, namely what is the (approximate) probability that the player will make 300 or more out of 400 attempts?
3. The number of defective items in a lot of electronic chips has a Poisson distribution with mean 4. A company has just received 10 such lots. If the company finds more than one defective item in any lot, it will return the lot to the manufacturer. Let X be the number of lots that will be returned to the manufacturer. Find the mean and variance of X .
4. A particle is moving along the x axis. At each step it either moves one unit to the right or one unit to the left with probabilities $1/4$ and $3/4$ respectively. Suppose that the particle is at 0 initially. What is the probability that it will be at $x = 2$ after 8 such steps?
5. Let $X \sim \text{Uniform}[0, 2]$. Find the expected value of $Y = \ln(X)$.
6. The lifetime distribution of my car has a hazard rate $\lambda(t) = \frac{t}{100}$. Currently my car is ten years old. What is the probability that my car will fail within a year?
7. A continuous r.v. X has the following c.d.f.

$$F(x) = 1 - e^{-30x^4}, \quad x \geq 0.$$

Let $Y = 3X^4$. Write the c.d.f of Y . Give the name and the parameter(s) of the distribution of Y ?

8. Automobile accidents occur in a certain town at an average rate of 4.2 accidents per week. Making suitable assumptions find the probability that there will be more than two accidents tomorrow.
9. Suppose that a random variable has a probability density function given as:

$$\begin{aligned} f_Y(y) &= 0; \quad y < 1 \\ &= \frac{1}{4}; \quad 1 \leq y < 2 \end{aligned}$$

$$= \frac{3}{10}y; \quad 2 \leq y \leq 3$$

$$= 0; \quad y > 3.$$

Find the cumulative distribution function.

10. A husband and wife invest their \$ 2000 IRAs in two different portfolios. After one year the husband's portfolio has .2 probability of losing \$200 and a .8 probability of gaining \$400. The Wife's portfolio has 0.1 probability of losing \$100 and .9 probability of gaining \$300. Find the standard deviation of the *difference of gains* of the husband and wife.

11. Let $T \sim Exp(\lambda)$. Let X be a discrete random variable defined as $X = k$ if $k - 1 \leq T < k$; $k = 1, 2, \dots$. Write the probability mass function of the discrete random variable X . What is the name of the distribution of X ?

12. In each of the following say what is the name of the distribution of X and what are the parameters of the distribution.

(a) Buses arrive on a bus stop at a rate of 10 per hour. X represents the number of buses that will arrive in the next 10 minutes.

(b) Each bus that arrives on a bus stop is (independently of other buses) a bus that goes to downtown with probability .7. Let X be the number of buses I will have to see in order to get one that goes to downtown.

(c) Let X represent the number of buses, out of the next 25 buses that arrive, that are going to downtown.

(d) I will play a game in a casino until I lose four times. Suppose that my chance of winning on any game is .45. X represents the number of games I play before I leave.

(e) X is a continuous non negative random variable whose Hazard rate function is $\lambda(t) = 3$ for all $t \geq 0$.

(f) A dart is thrown randomly at a circular dart board of radius 10. Assuming that the dart will hit the board, let X denote the angle the ray joining the center of the board and the point of hit makes with the horizontal axis.