Statistics 164 Syallabus

1 Probability Preliminaries

a. Probability model for random experiment
   - Basic properties of probabilities
   - Definition of random variables
   - Basic properties of CDF's

b. Absolutely continuous and discrete r.v.
   - pmf's and densities
   - examples
   - joint distributions
   - independence
   - expected values
   - moments and covariance

2 Distribution of functions of r.v.

a. The CDF method
b. Convolutions and basic properties
c. Change of variables*

3 Derivation of common univariate distributions

a. Double exponential
b. Definition and basic properties of gamma integral
c. Gamma distributions and properties
d. Beta distributions and properties
e. Chi-squared distributions and properties
f. F and T distributions

4 Miscellaneous

a. Probability and probability inverse transformations
b. Order statistics
c. Stirling's formula*
d. Moment generating functions*
e. Poisson approximation to binomial via coupling

5 Convex functions and inequalities

a. Definition and basic properties* of convex functions
b. Jensen's inequality
c. Hölder's inequality
d. Lyapounov's inequality
e. Minkowski's inequality

6 Probability inequalities

a. Markov and Chebyshev inequalities
b. Chernoff bound
c. Hoeffding's inequality
d. Bennett and Bernstein inequalities
e. McDiarmid's (bounded difference) inequality*

7 Conditional distributions and expectations

a. Basic definitions and properties (discrete case)
b. Examples

8 Information theory preliminaries

a. Definition and basic properties of entropy
b. Definition and basic properties of relative entropy

9 Convergence in probability

a. Definition
b. Continuous and uniformly continuous functions
c. Closure properties of in-probability convergence
d. Weak law of large numbers

10 Convergence in distribution

a. Definition (using expectations of bounded continuous functions)
b. Some examples
c. Equivalent definitions*
d. Basic properties
   - connections with convergence in probability
   - continuous mapping theorem (map everywhere continuous)
e. Slutsky's theorem

11 Big O_p and little o_p notation

a. Definitions
b. Basic properties

12 Characteristic functions and the CLT

a. Integrating complex functions
b. Characteristic functions and basic properties*
c. Uniqueness, continuity, and inversion of Chf's*
e. Statement of CLT and proof in case $E|X|^3 < \infty$.
f. Some applications of CLT

13 The delta method

a. Delta method for weak convergence
b. Delta methods for moments and some corollaries

14 Multivariate analysis

a. Orthogonal transformations and independent normals
b. Matrix preliminaries and review
c. Expectation and covariance of random vectors
d. Multivariate normal (non-singular case)
   - density
   - geometry
   - definition and basic properties* of multivariate Chf's
   - multivariate CLT and delta method*
e. Multinormal distribution (general case)
   - definition and basic properties
   - independence of jointly multinormal r.vectors
   - linear transformation of multinormals
   - conditional distributions of jointly multinormal r.vectors

15 Quadratic forms in multinormal r.vectors

a. Definition and basic properties of non-central chi-squared
b. Quadratic forms and idempotent matrices
c. Distribution of QF's of idempotent matrices
d. Independence of linear and quadratic forms of multinormals