Homework 3 (Stor 635)

The due date is February 19 (Tuesday), 2008. The homework assignment contains 8 problems concerning convergence (almost sure, in probability, in distribution) and related notions (such as tightness). Individual work is encouraged but you may also discuss the problems with your classmates or me. Good luck!

Problem 1. Exercise 11.3 from Lecture Notes.

Problem 2. Exercise 11.6 from Lecture Notes.

Problem 3. Exercise 11.10 from Lecture Notes.

Problem 4. Exercise 11.12 from Lecture Notes.

Problem 5. (i) Exercise 11.17 from Lecture Notes. If the hint in Lecture Notes is not helpful, google or look for Scheffé's Lemma for extra help. (ii) If $\xi_n = \text{Normal}(\mu_n, \sigma_n^2)$ and $\xi = \text{Normal}(\mu, \sigma^2)$ are normal random variables, show that $\xi_n \to_d \xi$ if $\mu_n \to \mu$ and $\sigma_n \to \sigma$.

Problem 6. (i) Let μ_n, μ be probability measures on a countable set $D = \{a_j\}_{j\geq 1} \subset \mathbb{R}$. Let $p_{nj} = \mu_n(\{a_j\})$, $j \geq 1$, $n \geq 1$ and $p_j = \mu(\{a_j\})$. Show that, as $n \to \infty$, $\mu_n \to_d \mu$ iff for all j, $p_{nj} \to p_j$ iff $\sum_j |p_{nj} - p_j| \to 0$. (ii) Let ξ_n be Binomial(n, p_n), $n \geq 1$, random variables. Suppose $np_n \to \lambda$, $0 < \lambda < \infty$. Show that $\xi_n \to_d \xi$, where ξ is a Poisson(λ) random variable.

Problem 7. Consider some random variables ξ_n and η_n such that $\{\xi_n\}$ is tight and $\eta_n \to_P 0$. Show that $\xi_n \eta_n \to_P 0$.