Homework 5 (Stor 635)

The due date is March 6 (Thursday), 2008. The homework assignment contains 7 problems concerning characteristic functions, their basic properties, connections to moments, and convergence in distribution. Individual work is encouraged but you may also discuss the problems with your classmates or me. Good luck!

Problem 1. Exercise 12.4 from Lecture Notes.

Problem 2. Exercise 12.10 from Lecture Notes.

Problem 3. Let ξ be a random variable and ϕ be its distribution function. If $0 < \alpha < 1$ and $E|\xi|^\alpha < \infty$, show that $\phi(t) - 1 = o(|t|^\alpha)$ as $t \to 0$. For $1 \le \alpha < 2$, show the same result under additional assumption that $E\xi = 0$. (Hint: the case $1 \le \alpha < 2$ is harder. Consider the real and imaginary pats of $\phi(t) - 1$ separately and write the latter as

$$E\sin(t\xi)1_{\{|\xi|\le\epsilon/t\}} + E\sin(t\xi)1_{\{|\xi|>\epsilon/t\}}.$$

The second is bounded by $(|t|/\epsilon)^\alpha E|\xi|^\alpha 1_{\{|\xi|>\epsilon/t\}} = o(|t|^\alpha)$ for fixed ϵ. In the first expectation use $\sin(tx) = tx + O(|tx|^3)$.

$$Et\xi 1_{\{|\xi|\le\epsilon/t\}} = -tE\xi 1_{\{|\xi|>\epsilon/t\}}, \quad E|t\xi|^3 1_{\{|\xi|\le\epsilon/t\}} \le \epsilon^{3-\alpha} E|t\xi|^\alpha.)$$

Problem 4. Exercise 12.17 from Lecture Notes.

Problem 5. Exercise 12.19 from Lecture Notes.

Problem 6. Exercise 12.20 from Lecture Notes.

Problem 7. Exercise 12.21 from Lecture Notes. (The first part of the problem appears in earlier homework. Do it here using characteristic functions.)