

# Nonlinear Equations and Optimization

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# Nonlinear Equations

$$F(x) = 0, x \in R^N$$

- Common in simulation (PDE, ODE, ...)
- Many codes for standard problems.
- SAMSI interest: porous media flow and transport.

# Standard Problem + Standard Methods

- $F$  smooth,  $F(x^*) = 0$ ,  $F'(x^*)$  nonsingular

- Newton's method

Data:  $x_c$  current iterate,  $F$

- Evaluate:  $F(x_c)$ , terminate?
- Solve  $F'(x_c)d = -F(x_c)$
- Find  $\lambda$  so that  $x_+ = x_c + \lambda d$  is useful.
- Repeat

# Other Stuff

How do you solve

$$F'(x_c)d = -F(x_c)?$$

Choices: Direct, Iterative

What if the standard assumptions fail?

They do in many problems.

# Optimization

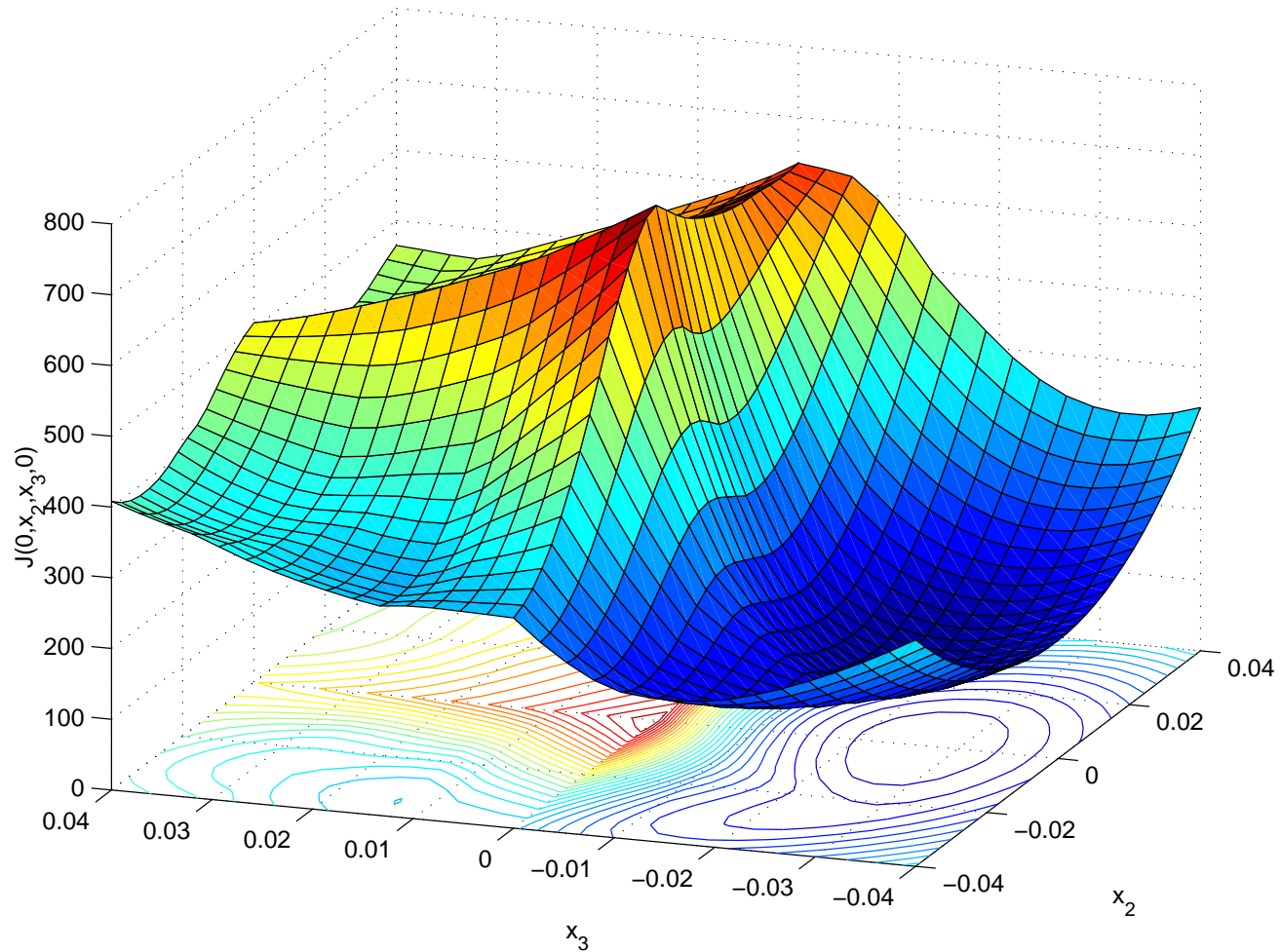
Problem:  $\min_{x \in D} f(x)$

- Gradient-based if  $f$  and  $D$  are smooth.
- Sampling for nonsmooth.
- Not much problem if  $f$  is unimodal and  $D$  is simple

# Problems

- Multimodal  $f$ ; global optimization
- Hidden constraints
- Randomness in  $f$  or  $D$

# Optimization Landscape I



# Optimization Landscape II

