STOR 890, Spring 2011, Homework 1

Note: Try these problems yourselves and contribute to the class discussion on Tue. 2/22.

(1) A combined spot/futures market contains a bank account $B$, a stock $S$ and a futures contract $FU$ related to $S$. Suppose the following data are collected:

$$S(0) = 1, \quad S(1) = 1.02, \quad S(2) = 0.99, \quad S(3) = 1.01, \text{ (unit price in dollar);}$$

$$FU(0, 3) = 1.04, \quad FU(1, 3) = 1.01, \quad FU(2, 3) = 1.02, \text{ (unit price in dollar);}$$

$$B(0) = 1, \quad r(1) = 0.001, \quad r(2) = r(3) = 0.002.$$  

(1a) How much does it cost to take a long position of $FU$ (with 100 shares) at $t = 1$?

(1b) For a short position of $FU$ (with 100 shares) taken at $t = 2$, what is the payoff at $t = 3$?

(1c) Frank starts with $1,000 in $B$. Suppose at $t = 1$, he shorts 100 shares of $S$ and takes a long position of $FU$ with 50 shares and the maturity date $T = 3$. What is the gain $G(t)$ in Frank’s portfolio at $t = 2$ before marking to market? What is the value $V(t)$ of his portfolio at $t = 3$?

(2) Construct an example of arbitrage-free but incomplete single period model.

(3) Consider the binomial tree model: $T = 3$, $u = 1.07$, $d = 0.92$, $r = 6\%$ and $S(0) = 2$ (see the example in Lecture 2). Suppose a constant dividend yield $\lambda = 5\%$ of the stock price is issued at the ex-dividend date $t = 2$.

(3a) For the chooser option (see Example 4.3) with the decision time $T_0 = 2$, the expiry $T = 3$, and the exercise price $c = 2.05$, construct a replicating portfolio and represent it in a binomial tree similar to Figure 2.3.

(3b) Calculate the value process of the American chooser option with the decision time $T_0 = 1$ and the exercise price $c = 2.05$.

(3c) Determine the optimal exercise strategy.

(4) Assume the stock price process starts with $S(0) = 1$ and follows a binomial tree with $u = 1.08$, $d = 0.91$. Moreover, the short rate process is the same as in Example 7.1.

(4a) Consider a forward contract and a futures contract on the stock, both start at $t = 0$ and end at $T = 3$. Do they have the same value at $t = 0$? Do they have the same price? Why?

(4b) For a European call option and a European put option on the futures $FU(1, 3)$, specify the exercise price $c$ such that these two European options have the same value at $t = 0$. 
