Simple Hedging (Part 1)

890-2011

2/3/2011
Hedging: to reduce the risk in an invested security by holding another security with possible “opposite” scenarios.

Diversification between negatively correlated stocks is an example of hedging.

not a free lunch, often at the expense of reduced returns, similar to buying an insurance ... more sophisticated examples: CDO, CDS (later).
Consider investment options at a beach based on the following annual return table:

<table>
<thead>
<tr>
<th></th>
<th>umbrella maker (U)</th>
<th>resort owner (R)</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>rainy</td>
<td>50%</td>
<td>-25%</td>
<td>0.5</td>
</tr>
<tr>
<td>sunny</td>
<td>-25%</td>
<td>50%</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Compare three portfolios:

- $P_1$: all wealth invested in U
- $P_2$: all wealth invested in R
- $P_3$: wealth split equally between U and R
Note:

- $P_1$, $P_2$ and $P_3$ have the same expected return $\mu = 12.5\%$.
- $P_1$ and $P_2$ have the same risk, i.e. their variance

$$
\sigma^2 = 0.5 \times (50\% - 12.5\%)^2 + 0.5 \times (-25\% - 12.5\%)^2
$$

$$
= 0.5 \times (37.5\%)^2 \approx 0.07.
$$

- However, $P_3$ has no risk (its variance equals zero) because it consists of equal shares of $U$ and $R$, i.e. $c_1 = c_2 = 1/2$, hence the return for $P_3$ is constant

$$
\frac{1}{2} \times (50\%) + \frac{1}{2} \times (-25\%) = 12.5\% \text{ regardless of the weather at the beach!}
$$
Issues:

- Why Case 1 could be regarded as a caricature of “bull market”?

- In an opposite situation, if both $P_1$ and $P_2$ have negative expected returns, i.e. $\mu_1 < 0$ and $\mu_2 < 0$ (see Case 2 as an example), then no matter what proportions $c_1$ and $c_2$ between $U$ and $R$ we set, the resulting portfolio will always have a negative expected return. Why?

- In the above “bear market”, what could we do to come up with a profitable portfolio?
Consider a setting similar to Case 1 except ...

<table>
<thead>
<tr>
<th>umbrella maker (U)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>rainy</td>
<td>25%</td>
<td>-50%</td>
</tr>
<tr>
<td>sunny</td>
<td>-50%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Compare three portfolios:

- $P_1$: all wealth invested in $U$
- $P_2$: all wealth invested in $R$
- $P_3$: wealth split equally between $U$ and $R$
- $P_4$: another choice?
Case 2 (continued)

Summary:

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>-12.5%</td>
<td>-12.5%</td>
<td>-12.5%</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.07</td>
<td>0.07</td>
<td>0</td>
</tr>
</tbody>
</table>

- All three portfolios are no good, but $P_3$ is the worst! Why?
- Any hope if shorting is allowed?
- $P_4 = (c_1, c_2)$: assume $c_2 < 0$ (short R), and $c_1 + c_2 = 1$.
- Still, $P_4$ always has a negative expected return because

$$
\mu_4 = c_1 \mu_1 + c_2 \mu_2 = (c_1 + c_2) \mu_1 = \mu_1 = -12.5%.
$$
<table>
<thead>
<tr>
<th>umbrella maker (U)</th>
<th>resort owner (R)</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>rainy</td>
<td>10%</td>
<td>-50%</td>
</tr>
<tr>
<td>sunny</td>
<td>-30%</td>
<td>10%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>-10%</td>
<td>-20%</td>
<td>$-10% (1 + c_2)$</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.04</td>
<td>0.09</td>
<td>?</td>
</tr>
</tbody>
</table>

- $P_2$ is less profitable and more risky than $P_1$, but is it really worse?
- By shorting $R$, $P_4$ could turn profitable (i.e. $\mu_4 > 0$) if $c_2 < -1$.
- Caveat: The risk $\sigma_4^2$ is a quadratic function of $c_2$ with a positive coefficient for the term $c_2^2$. Implication? — It becomes more risky as you short more aggressively.
Simple Hedging (Part 2)

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Forward contract: an agreement by two parties to exchange some items in the future at a prearranged price.

Example: making a reservation on a flight and lock into an airfare

Terms: delivery price, spot price, forward price

Forward price: A contract starts at current time $t$ on a stock $S$ with price $S_t$; The buyer will purchase $S$ at future time $T > t$ with delivery price $F_t$, called the forward price, determined by

$$F_t = S_t (1 + r)^{T-t}, \quad \text{Why?}$$

where $r$ denotes a constant interest rate.

Derivatives: Forward is an example of derivatives, because it is a security derived from a basic security $S$. 
In each of the above examples, a forward contract plays a hedging role by reducing the risk.

Note: The spot stock price $S_T$ or the actual airfare at time $T$ becomes irrelevant in a forward contract.

Futures: a standardized forward contract that is traded on an organized exchange.

Why need standardization?
Forward vs futures

- **Trading**
  - Forward: over-the-counter (OTC)
  - Futures: on exchanges, e.g. CBOT, NYFE

- **Standardization**
  - Forward: not standardized, everything based on private negotiation
  - Futures: standardized by specifying what to be traded, price, delivery date and location, etc.

- **Delivery**
  - Forward: usually taking place
  - Futures: A majority of contracts are closed out prior to maturity (how?)
Forward vs futures (continued)

Collateral and Margin
- Forward: collateral negotiable, no daily adjustment before final settlement, default risk exists ...
- Futures: trades regulated by an exchange, margin account required similar to trading stocks, daily adjustment referred to as marking to market depending on the difference between the spot price on that day and the predetermined futures price

Information and Liquidity
- Forward: volume information unavailable, low liquidity (due to great variability between contracts)
- Futures: volume information published, high liquidity due to standardization
Example: futures in commodity trading

- A farmer in Kansas wants to sell 100,000 bushels of wheat to a baker in NYC.
- Both of them enter a futures contract 1 month before harvest time, taking long and short positions respectively in the contract.
- Forward price (or futures price): $2 per bushel
  Note: Forward price and futures price need not be the same in general.
- Futures market can implement the deal conveniently.
- What transactions would take place on delivery date T?
- Consider possible spot prices $1.5, $2, $2.5 at time T...
Hedging vs Insuring

- **Hedging**: eliminate (reduce) the risk of loss by giving up the opportunity gain
- **Insuring**: pay a premium to eliminate the risk of loss and retain the potential for gain
- **Example**: making flight reservations...
  - hedging: lock into an airfare (say $500 to London) now with no fees (same as a forward contract)
  - insuring: pay $20 fees now for a right but not an obligation to buy a ticket to London around Christmas at $500; if the spot fare > $500, buy the ticket, otherwise do nothing; so $20 is a premium to insure your total cost would not exceed $520 — this is a call option written on the airfare.
Call option in the example of air ticket:
- Expiration date: predetermined in the contract, the day to decide whether to exercise
- Strike price: $500
- Underlying asset: an air ticket to London
- Option price: $20 ... How to determine it? option pricing — a big task.

Put option: a right to sell something
Put option on Microsoft stock

Motivation: You anticipate a fall from $27 now to $24 by the end of 2010.

What to do: You enter a contract (put option) with the expiration date 12/30/2010, to earn a right for choosing between selling 500 shares of MSFT (you own now) to the counter party at $27 per share and doing nothing.

Option price: $2 on a single share ...

Possible scenarios: on 12/30/2010, MSFT could be $30, or $27, or $24, evaluate possible gain/loss ...

A portfolio (for hedging) that combines an above put option and 500 shares of MSFT.

Why need it?
Consider the portfolio that consists of 500 shares of MSFT stock and the above put option.

Pay-off table for the portfolio: For simplicity, we leave out the fixed number 500 shares and just compare portfolio values per share at $t$ (now) and at $T$ (12/30/2010) from the perspective of the put option holder.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$30</th>
<th>$27</th>
<th>$24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value at $t$</td>
<td>27-2=25</td>
<td>27-2=25</td>
<td>27-2=25</td>
</tr>
<tr>
<td>Value at $T$</td>
<td>30 (not sell)</td>
<td>27 (not sell)</td>
<td>27 (exercise)</td>
</tr>
</tbody>
</table>